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The detection efficiency of electron multipliers

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Abstract. The variation of the detection efficiency of electron multipliers used in the pulse-counting mode is investigated as a function of the average gain per dynode, the energy of the electrons being detected and the applied voltage. It is assumed that the pulse-height distributions can be obtained from Polya statistics and that the universal yield curve for secondary emission is applicable. Results are presented for 12-stage multipliers having values of the Polya parameter $b=0{\cdot}0$, $0{\cdot}5$ and $1{\cdot}0$.

1. Introduction

A theoretical investigation of the efficiency with which bare electron multipliers detect electrons when used in the pulse-counting mode is made in this paper. The work has its origin in the recent application of multipliers to measuring flux densities and energy spectra of low-energy radiation in space, but the results should have wider usefulness. These results are given in terms of the average gain per dynode μ, the particle energy E_{M} or dynode potential V_{M} at which this average gain is maximum, and the Polya parameter b. The latter parameter, which is discussed in detail by Prescott (1966), determines the shape of the pulse-height distribution at the multiplier anode.

The type of distribution characterized by a value of b near unity has been interpreted in terms of a broad distribution of local values of the secondary emission coefficient across the dynode surface (Baldwin and Friedman 1965). Thus multipliers that focus the electron cascade from dynode to dynode have values of b very close to zero, while those in which the cascade becomes broad and diffuse tend to have b closer to unity. It can be expected then, that box-and-grid and venetian-blind multipliers have a high b value, while the RCA type, in common with other focused types, has a low value. However, this has not yet been subjected to systematic experimental investigation, and can in any case be only a tendency to high or low values since the condition of the dynode surface also affects the value.

Detection efficiencies are obtained by integrating computed pulse-height distributions over all pulse heights greater than a minimum value which is determined by the discriminator level of the anode pulse amplifier. Although pulse-height distributions vary markedly between different types of multiplier, most of the observed distributions can be represented reasonably well by using appropriate values of μ and b. In addition, it is found that the secondary yields of many substances as a function of primary energy, when normalized suitably, lie on the same curve—the universal yield curve. These facts make possible the general discussion of electron multiplier detection efficiency that is given here.

2. Pulse-height distribution

In deriving the pulse-height distributions it is assumed that the creation of secondary electrons at a dynode when a primary impinges on it is governed by Polya statistics (Arley 1943). Thus, if at time t n secondaries have been produced, the probability that a further secondary will be produced in the interval $(t, t+\Delta t)$ is

$$\lambda \frac{1+bn}{1+b\lambda t}\,\Delta t+\mathrm{o}(\Delta t) \tag{1}$$

where λ and b are non-negative constants. This leads to the Polya distribution, according to which the probability for producing n secondaries is

$$P(n)=\frac{\mu^n}{n!}(1+b\mu)^{-n-1/b}\prod_{i=1}^{n-1}(1+bi). \tag{2}$$

When $b=0$ the distribution is Poissonian, and when $b=1$ it is the monotonic decreasing Furry distribution. The mean of the distribution is independent of b and is given by $\mu=\lambda t$, which we identify also as the average gain of the secondary emission process.

If the Polya process is applied to each of the dynodes successively after the method of Lombard and Martin (1961), then after k stages the frequency distribution for the total number of secondaries n is (Prescott 1966)

$$P_k(n)=\frac{\mu}{n}P_k{}^b(0)\left(\sum_{i=0}^{n-1}(n+ib-i)\,P_k(i)\,P_{k-1}(n-i)\right), \qquad k\geqslant 1,\ n\geqslant 1 \tag{3}$$

where $P_k(0)$ is the probability for the total number of secondaries to be zero at the kth stage (blank cascade) and, when b is not zero, is obtained from

$$P_k(0)=\{1+b\mu(1-P_{k-1}(0))\}^{-1/b}. \tag{4}$$

When $b=0$ the probability for the cascade to be blank at the kth stage is

$$P_k(0)=\exp(\mu P_{k-1}(0)-\mu). \tag{5}$$

Since the response to a single input electron is required, the computation is started with the initial condition

$$P_0(n)=\delta_{1n}. \tag{6}$$

On calculating the pulse-height distribution at successive dynodes from equations (4), (5) and (6) it is found that the distributions converge. For k such that $\mu^k>100$, the distribution will have reached this final form for practical purposes. The mean of the distribution at any stage is the product of the average gains of the stages up to and including that being considered. Since a uniform average gain of μ has been assumed at each stage, the mean of the distribution at the lth stage is μ^l. Thus the normalized distributions $\mu^l P_l(n\mu^{-l})$ will be identical for all stages $l\geqslant k$. This allows the computation of the distribution function to be terminated at $k=3$ for $\mu=5$, and $k=4$ for $\mu=4{\cdot}3$, and so on. This is important since the computations are very time consuming. Examples of the pulse-height distributions for $\mu=5$ and for Polya parameter values $b=0$, 0·5 and 1·0 are shown in figure 1.

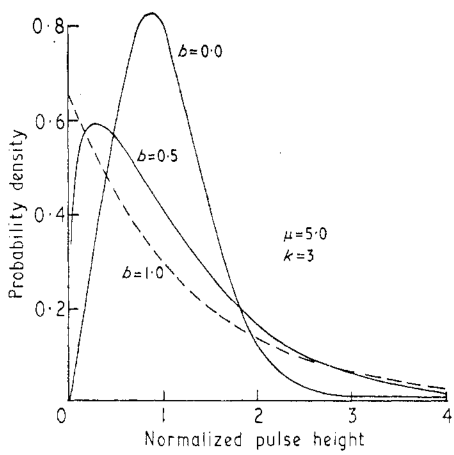

Figure 1. Pulse-height distributions for an electron multiplier having an average gain per dynode $\mu=5$. The mean of the distributions is unity.

3. Universal yield curves

The analysis by Baroody (1950) of the dependence of secondary emission on the energy of the primary electron has shown that a universal yield curve exists. The existence of such a curve is confirmed by elementary theories of secondary emission, although the fit between the observed and the theoretical curves is not good at energies above E_M, especially for metals.

The theory assumes that the range of primaries in the secondary emitter is proportional to the square of the primary energy, leading to Whiddington's law for the energy loss of the primaries per unit path length (Dekker 1954). However, measurements by Young (1956) on Al_2O_3 show that the range varies as $E^{1 \cdot 35}$ for primary electrons up to 5 kev. In order to get a better fit to the observed universal yield curve, it is assumed here that the range of the primaries is given by

$$R = cE^s \tag{7}$$

where the exponent s is to be adjusted to give the best possible fit to Baroody's experimental yield curve.

Following the usual analysis, the secondary yield is

$$\mu = \int n(x)\, g(x)\, dx \tag{8}$$

where $n(x)\,dx$ represents the number of secondaries produced by one primary at a depth between x and $x + dx$ below the dynode surface, and $g(x)$ represents the probability for such secondaries to escape from the surface. If it is assumed that

$$n(x) = \gamma\, dE/dx \tag{9}$$

and that the escape probability is governed by an exponential process

$$g(x) = g_0 \exp(-\alpha x) \tag{10}$$

then the expression for the yield becomes

$$\mu = c^{1/s} s^{1/s-1} \gamma g_0 \int_0^{x_p} \frac{\exp(-\alpha x)}{(x_p - x)^{1-1/s}}\, dx \tag{11}$$

where x_p is the maximum penetration of the primaries into the dynode, and is given by

$$x_p = E^s/sc. \tag{12}$$

Introducing

$$y^s = \alpha(x_p - x) \tag{13}$$

and

$$z = (\alpha x_p)^{1/s} \tag{14}$$

equation (11) becomes

$$\mu = \left(\frac{sc}{\alpha}\right)^{1/s} \gamma g_0 \exp(-z^s) \int_0^z \exp(y^s)\, dy. \tag{15}$$

At the primary energy E_M for which the maximum yield μ_M is observed, equations (12) and (14) give

$$E_M = z_M (sc/\alpha)^{1/s} \tag{16}$$

so that

$$\frac{\mu}{\mu_M} = \frac{F(z)}{F(z_M)} = \frac{F(z_M E/E_M)}{F(z_M)} \tag{17}$$

where

$$F(z) = \exp(-z^s) \int_0^z \exp(y^s)\, dy. \tag{18}$$

Thus

$$dF/dz = 1 - s z^{s-1} F(z) = -f(z). \tag{19}$$

The root of $f(z)$ will be the value of z for which $F(z)$ is a maximum. This root was found iteratively by the Newton–Raphson method:

$$z_{i+1} = z_i - \frac{\beta s F(z_i) - 1}{s[F(z_i)\{(s-1)\beta/z_i - s\beta^2\} + \beta]} \tag{20}$$

where $\beta = z_i^{s-1}$ and z_i is the ith approximation to z_M.

The yield μ/μ_M as a function of E/E_M was computed from equation (17), using (18) for the evaluation of $F(z)$ and (20) to determine z_M. The integral in (18) was computed to a relative accuracy of 1 in 10^4, and the iteration in (20) was terminated when z_M had been determined to a relative accuracy of 1 in 10^5. This was done for a range of values of s. It was found that $s = 1{\cdot}14$ fitted Baroody's data for metals very well, as can be seen from figure 2. Also plotted in this figure is the curve for $s = 2{\cdot}0$, since the data of Johnson and McKay (1953) indicate that the yield of MgO follows this curve approximately.

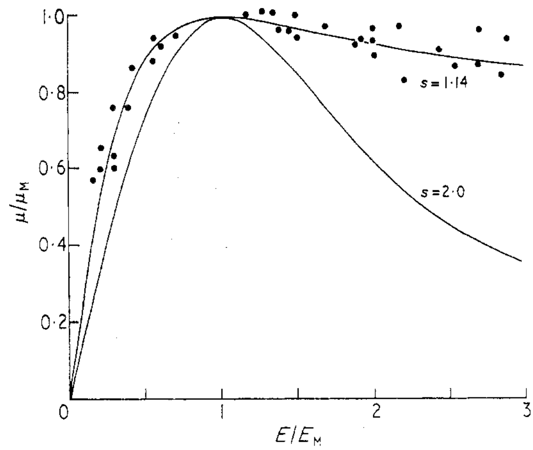

Figure 2. Universal yield curves. The points are from Baroody (1950), the full lines from equation (17).

4. The detection efficiency

4.1. *Variation with dynode gain*

The absolute detection efficiency can be obtained by integrating pulse-height distributions like those in figure 1, but this quantity will have little practical significance other than setting a limit to the maximum attainable efficiency for a particular multiplier. To avoid counting a significant number of noise pulses it is necessary to establish a discrimination level. If the discriminator of the anode pulse amplifier is set so that only those pulses formed from cascades of N or more electrons are detected, then the detection efficiency of a multiplier with m stages, each with an average gain μ, is

$$\epsilon(m, \mu, N) = 1 - \sum_{n=0}^{N} P_m(n). \tag{21}$$

Since the computation of the pulse-height distribution is performed out to the kth stage only, a relation between efficiencies at the kth and mth stages is needed in order to use (9). This can be obtained by remembering that the distribution has converged by the kth stage, so that the detection efficiency for the multiplier can be obtained from the kth stage distribution if the discriminator level is reduced from N to $N\mu^{-(m-k)}$:

$$\epsilon(m, \mu, N) = \epsilon(k, \mu, N\mu^{-(m-k)}) \tag{22}$$

so that

$$\epsilon(m, \mu, N) = 1 - \sum_{n=0}^{Nt} P_k(n) \tag{23}$$

where $t = \mu^{-(m-k)}$.

Pulse-height distributions $P_k(n)$ were computed for a number of values of μ ranging from 1·5 to 5·0. These distributions were used in equation (23) to obtain the detection efficiency of a 12-stage multiplier as a function of μ for discriminator levels $N=10^3$, 10^4, 10^5, 10^6, 10^7 and 2×10^7. This was done for three values of the Polya parameter, $b=0{\cdot}0$, 0·5 and 1·0. The results, displayed in figure 3, show that the detection efficiency can vary very rapidly with μ over certain ranges of the gain, especially for a multiplier with a low b. It is desirable to operate the multiplier on the flatter part of these curves; thus they should be useful in determining the optimum multiplier operating conditions. These results apply to a 12-stage multiplier, but it is easy to obtain similar curves for a multiplier with any number of stages from equation (23) if the $P_k(n)$ are known for different values of μ. These are expensive to compute but can be obtained from the author for the range of μ mentioned above, and for $b=0{\cdot}0$, 0·5 and 1·0.

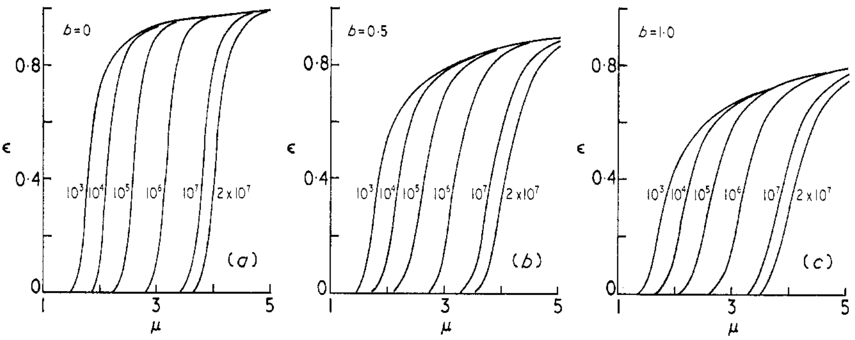

Figure 3. Relationship between the detection efficiency ϵ and the average gain per dynode μ for three 12-stage electron multipliers having $b=0{\cdot}0$, 0·5 and 1·0. The curve parameters are discriminator levels, given in terms of number of electrons.

4.2. *Detection efficiency spectra*

If a bare multiplier is used to measure fluxes of particle radiation, then it is essential that the detection efficiency be known as a function of the energy of the incoming radiation. This relationship can be derived from the universal yield curve which determines the variation of the gain of the first dynode with the radiation energy. Pulse-height distributions were computed with $\mu=3$ for all stages above the first, and with μ_1, the gain of the first dynode, ranging from values less than unity up to 5·0. Taking $\mu_M=5{\cdot}0$, the μ_1 values were transformed to E/E_M with the aid of figure 2, using the universal curve for which $s=1{\cdot}14$. Efficiencies were determined from equation (23) as before, and the resulting spectra are shown in figure 4 for the case $b=1{\cdot}0$. These calculations were repeated for a multiplier

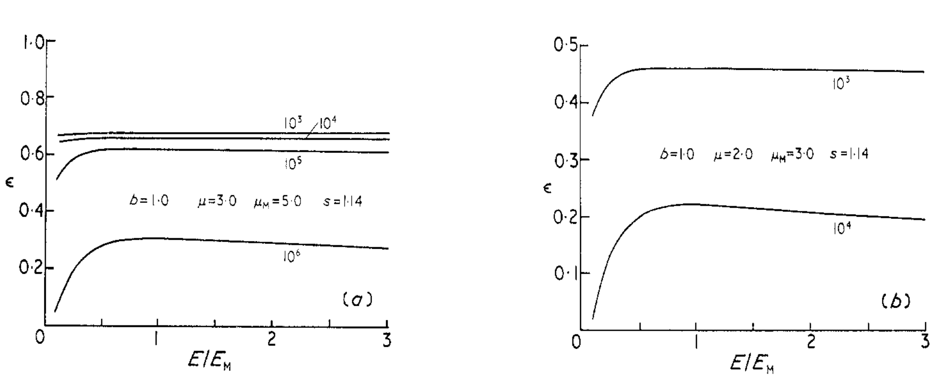

Figure 4. Detection efficiency spectra for a 12-stage multiplier having $b=1{\cdot}0$, assuming a universal yield curve characterized by $s=1{\cdot}14$. The curve parameters are discriminator levels.

with an average gain of 2·0 and $\mu_M = 3{\cdot}0$, and the results also displayed in figure 4. It is seen that the experimental conditions can be arranged so that the detection efficiency is practically independent of energy over all except the very lowest energies. This of course simplifies the calibration of the radiation detection equipment as well as the analysis of data obtained.

Since some substances display a gain–energy relationship characterized at the highest energies by $s=2$, efficiency spectra corresponding to those shown in figure 4, but with $s=2{\cdot}0$, were computed, and are shown in figure 5. Here the variation with energy cannot be ignored. Also plotted in figure 5 are experimental results (Wright 1967) obtained for a 12-stage AgMg multiplier, for which it is estimated that $b=1$, $\mu=2{\cdot}5$, and $N=10^5$. It is seen that the AgMg multiplier, which has $E_M = 800$ ev, has a high-energy efficiency dependence similar to the $s=2{\cdot}0$ curves.

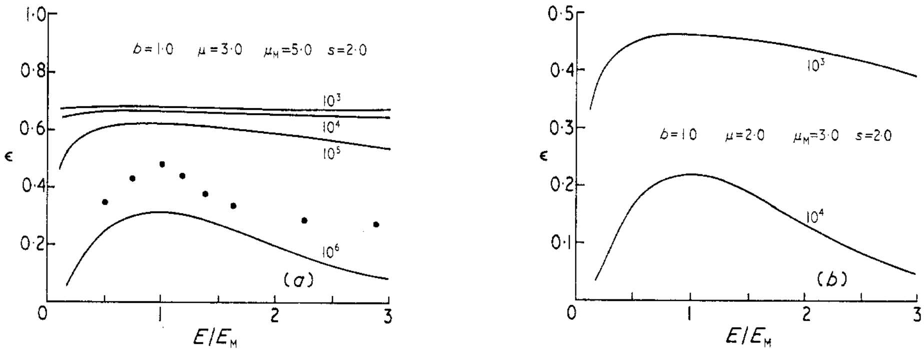

Figure 5. Detection efficiency spectra for a 12-stage multiplier having $b=1{\cdot}0$, assuming a universal yield curve characterized by $s=2{\cdot}0$. The curve parameters are discriminator levels. The points are experimental (Wright 1967).

4.3. *Variation with applied voltage*

It is possible to combine the universal yield curve with the data of figure 3 to give the detection efficiency of the multiplier as a function of the applied voltage, since the energies of secondaries as they impinge on successive dynodes are determined essentially by the potential difference between dynodes. Equation (17) with $s=1{\cdot}14$ was used to obtain

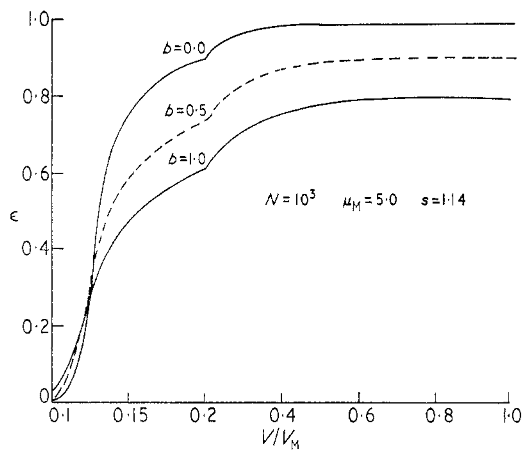

Figure 6. Detection efficiency of a 12-stage multiplier as a function of the applied voltage/stage. There is a change of scale at $V/V_M = 0{\cdot}2$.

μ/μ_M at values of E/E_M spaced 0·01 apart, for E from 0 to E_M. Assuming a maximum gain per dynode $\mu_M = 5{\cdot}0$, μ was obtained in terms of E/E_M, and figure 3 used to convert this to detection efficiency as a function of the applied voltage per dynode expressed as V/V_M.

The results for a discriminator level $N = 1 \times 10^3$ are shown in figure 6, with the Polya constant b as parameter. As the discrimination level is increased these curves shift to the right. The strong dependence of the detection efficiency on the dynode potential is very evident, especially when b is small. Since the measurement of the efficiency as a function of dynode potential is probably easier than a pulse-height analysis, it might provide a reasonably easy method for estimating the value of the Polya parameter for a particular multiplier.

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