THE MECHANISM OF CHANNEL ELECTRON MULTIPLICATION

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SUMMARY

The paper attempts to relate the important parameters observed in channelled electron multiplication to the physical processes involved. Ionic feedback and the effect of channel geometry on it are discussed. Saturation effects occurring in straight channels and curved channels are described and the different pulse height distributions are contrasted.

INTRODUCTION

The channel electron multiplier is now a well known instrument[1,2] which needs only a brief description before we discuss its operation in detail.

A channel multiplier is shown in Figure 1. It is a tube of suitably resistive material, with a length which is large compared to its diameter. Between the ends of the multiplying channel a potential of a few thousand volts is applied in vacuum. Electrons or other energetic particles striking the low voltage end of the channel release secondary electrons with some initial energy which carries them across the channel. While they traverse the channel the applied electric field accelerates the electrons axially until they collide with the wall with sufficient energy to release more secondaries. This process is repeated many times so that a very large number of electrons emerge from the output end. The pulse initiated by a single electron may in certain circumstances contain more than 10^9 electrons. The gain obtainable from a channel is a function of a number of parameters: the applied voltage, residual gas pressure, the resistance and geometry of the tube, and pulse repetition rate.

A channel multiplier amplifies direct currents and also counts energetic particles by producing corresponding pulses. This paper is devoted to pulsed operation.

THE GAIN-VOLTAGE CHARACTERISTIC

We can see intuitively that the energy of collision of secondary electrons with the channel wall will increase with the applied voltage but at the same time the number of collisions will decrease. We thus expect that the gain will rise to a maximum value and then decrease as the voltage is increased further. An expression showing the dependence of gain on the applied voltage is derived in Appendix I and is given in equation (1).

$$G = \left(\frac{KV_o^2}{4V\alpha^2}\right)^{\frac{4V\alpha^2}{V_o}} \quad \ldots \ldots \ldots (1)$$

where
- G is the gain
- V_o is the applied voltage
- V is the initial energy of the secondary electron
- α is the ratio of length to diameter
- K is a constant from the relation $\delta = KV_c$ where
- δ is the secondary emission coefficient
- and V_c is the collision energy

The theoretical curve in Figure 2 is a plot of equation (1) with reasonable values of the constants inserted. The experimental curve does not show a maximum, but rises to much higher gains than are predicted by the model and slowly flattens out as it approaches some saturation level. This is also shown in Figure 2. These discrepancies occur because the simplified theoretical model takes account only of diametric trajectories. In addition ionic feedback increases the gain very considerably in a channel multiplier.

IONIC FEEDBACK

In any practical system there is always some residual gas. Some of the gas molecules will be ionised by the electrons in the multiplied pulse as it approaches the output end of the tube. An expression for the probability of ionisation is derived in Appendix III. The resulting positive ions are accelerated back toward the input end where they may strike the wall and produce secondary electrons which re-start the gain process. This 'ionic feedback' enhances the gain of the channel very considerably.

Figure 3 shows the result on an oscilloscope of superimposing a large number of output pulses, each initiated by a single electron. The interval between the initiating pulse - the starter pulse - and the rest of the train gives an estimate of the charge to mass ratio of the relevant ion. The introduction of small amounts of argon and xenon into the chamber produces the results shown in Figure 4(a) and 4(b). The time intervals between the starter pulses and the prominent pulses of the trains correspond to A^+ and Xe^+, respectively accelerated over the full length of the channel. In addition, Figure 4(c) shows a feedback pulse which corresponds to Hg^+, presumably originating from the diffusion pump of the vacuum system. These results confirm that the multiple pulses are caused by ionic feedback. They also suggest that the first ion to strike the input end is a hydrogen ion.

Figure 5 shows the gain of the multiplier in both the starter pulse and the subsequent pulse train as functions of voltage. The interesting features of these curves are that the total gain in the pulse is about one or one and a half orders of magnitude greater than that in the starter pulse, and the gain of the starter pulse is not a function of pressure, whereas the total gain does increase with pressure. Figure 6 shows how the total gain varies with pressure at a given multiplier voltage.

Ionic feedback is a mixed blessing. We have seen that it increases the gain considerably, but makes it sensitively dependent on pressure. There are applications in which the pressure sensitivity is an embarrassment and it is then desirable to eliminate ionic feedback.

CURVED CHANNELS

The return of positive ions to the input end of the channel can be prevented by bending it. Evans[3] has calculated the minimum curvature necessary to eliminate feedback by making the ions collide prematurely with the wall. A curved channel produces a pulse at the output for each input particle as shown in Figure 7. The pulse is 40 nS wide in total.

The expression for the gain of a curved channel is derived in Appendix II and is given in equation (2).

$$G = \left\{ K \left(\frac{36\ V\ V_o^2}{\beta^2} \right)^{\frac{1}{3}} \right\}^{\left(\frac{V_o \beta^2}{36\ V} \right)^{\frac{1}{3}}} \quad \ldots \ldots (2)$$

where β is the ratio of the length of the channel to its radius of curvature, and the other parameters are the same as in equation (1).

The theoretical characteristic for a Mullard type B300A curved channel is shown in Figure 8, together with the experimental relation. There is good agreement over several orders of magnitude until the channel starts to saturate.

It is shown also in Appendix II that for most common curved channel geometries the electrons do not traverse the diameter of the channel but simply make repeated collisions with the outer wall as shown in Figure 18. The significance of this result is that the diameter, and hence the length to diameter ratio, is not important in a curved channel. The important parameter is β, the included angle of the curve. This has been verified by operating a curved channel multiplier having a value of β of 5.2, and a length to diameter ratio, α, of 18. The gain-voltage characteristic for this multiplier is shown in Figure 9, and a photograph of the multiplier is shown in Figure 10.

It has not been possible to operate straight tubes with such a low value of α. A straight channel with α = 28 has been operated to give a maximum gain of only 4×10^4.

SATURATION EFFECTS

The saturation effects seen in both straight and curved channels are not accounted for by the simple theories of channel multiplication. The two most probable causes are field distortion and space charge. The pulse in the curved channel is quite short in time and the maximum gain is less than in the straight channel; this suggests that pulses from curved channels are not limited by the same mechanism as those from straight channels.

Saturation in Curved Channels

The mechanism of total pulse saturation suggested by Evans[3] is that the emission of electrons from near the end of the channel raises the potential of the channel wall until there is a low field region which either maintains but does not augment the gain process or actually acts as an electron sink. This situation is analogous to that in a conventional photomultiplier when a very large light pulse reduces the interstage voltages near the output end so that the secondary emission coefficients are reduced. As with the photomultiplier it is possible to raise the limit on pulse height by connecting shunt capacitors across the last few stages. This is done with a channel multiplier by placing an earthed sleeve over the last centimetre or so. This sleeve also has the effect of broadening the pulse height distribution.

The pulse height distribution is a function of pulse repetition frequency. At high frequencies the distribution is broad; at low frequencies it is narrow. The height of a pulse also depends on the time interval since the preceding pulse; a pulse following quickly after another one will be small. As the resistance of the channel is reduced the pulse height distribution begins to broaden at progressively higher frequencies and the recovery time between pulses falls. These phenomena are illustrated in Figures 11 and 12. For a multiplier of resistance $2 \times 10^{10}\Omega$, the recovery time is about 1 msec. If the resistance is 10^9 Ω the recovery time is about 30 μsec. These figures are consistent with a field distortion model where the equivalent circuit is a resistance capacity chain as shown in Figure 13.

Since the electron energies emerging from a channel multiplier range up to 100 eV, the effective capacity of the channel may be considered to be that of the final 100 volt section. Thus with 5 kV applied across a 5 cm long channel, the effective "last stage" is the final 1 mm. The capacity of a cylinder having the dimensions of such a multiplier is about 4×10^{-12} farads/mm. For a multiplier of total resistance 2×10^{10} Ω, the calculated recovery time would thus be 1.6 msec. and for a resistance of 10^9 Ω, the recovery time would be 80 μsecs. The charge in this capacity is 2.5×10^9 electrons, compared with the experimentally observed saturated gain of about 4×10^9.

Saturation in Straight Channels

There are several pieces of evidence to suggest that the saturation mechanism in curved channels is different from that in straight channels.

(i) Figure 14 shows the pulses from a curved channel. At low frequencies the heights vary over quite a wide range in a random fashion. Unlike the pulses in Figure 11, their height is not related to the time interval between them.

(ii) The maximum gain in curved channels appears to be about 2×10^8, compared with 4×10^9 in straight ones.

(iii) Placing a capacitative sleeve round a curved channel has no effect on the size of the maximum pulse.

Thus it seems unlikely that field distortion is limiting the gain of curved channel multipliers. Bryant and Johnstone[4] have considered the effect of space charge in channel multipliers. Their analysis is more applicable to straight multipliers than to curved ones, but the same general principles apply. The cloud of electrons in the channel increases in density until it is sufficient to drive secondary electrons back into the channel wall without their acquiring enough energy from the field to produce more secondaries. A state of dynamic equilibrium is reached when this happens. It follows from Gauss' Law that n, the number of electrons per cm length uniformly distributed within a cylinder which will produce a voltage depression V volts at the centre of the cylinder is given by $n = 7 \times 10^6$ V electrons/cm.

Few electrons emerge from the channel with energies exceeding 100 eV. We may presume therefore that the maximum collision energy in the channel is also 100 eV. Channels are invariably vitreous and since for most glasses the secondary emission coefficient is unity at about 50 eV, a reduction in collision energy by a factor 2 would inhibit the gain process. It can be shown that a voltage depression of about 2.8 times the emission energy is required to achieve this reduction. This would suggest a maximum charge density of about 2×10^7 electrons/cm for 1 volt electrons. The output pulse persists for about 4×10^{-8} seconds, and since the mean energy is about 50 eV, the space charge limited gain is about 3×10^8 electrons. It should be noted that this is not a function of tube diameter. This result accords well with experimental findings.

We can summarise by saying that in curved channels the short pulse is limited by space charge, while in straight channels the long pulse train is limited by changes of the potential of the channel wall.

PULSE HEIGHT DISTRIBUTION

We have already alluded to the pulse height distributions in straight and curved channels. We can now consolidate the discussion of these phenomena.

In straight channels the pulse height distribution at maximum gain is very narrow at low pulse repetition rates. Figure 15 shows the contrast between the spread in height of the starter pulses which did not initiate feedback and of the larger saturating pulses. It shows that as the multiplying voltage and therefore the gain is increased the proportion of starter pulses giving rise to feedback also increases.

As the pulse repetition rate rises, the interval between pulses becomes shorter than the recovery time of the multiplier. The height of a pulse therefore becomes a function of the time interval since the preceding one, and the pulse height distribution broadens.

In a curved channel the maximum pulse height is limited by space charge and this shows itself in the pulse height distribution. Figure 16 shows distributions obtained from a Mullard B300A multiplier at various applied voltages. The resolution of a distribution is defined as the ratio of the full width of the distribution at half peak height to the gain at the peak height. The resolution improves with gain in the range 10^5 to 10^7 but then worsens at about 10^8. The best resolution is about 0.45, which is to be compared with about 1.5 obtained in conventional multidynode multipliers. The resolution in a conventional multiplier is limited by the statistical variation of the gain at each stage. According to Lombard and Martin[5], a resolution of 1.5 implies a gain per stage of 5, whereas a resolution of 0.45 would require a gain per stage of 25. Furthermore the variation of gain at each collision in a channel will be greater than that in a conventional multiplier, so an even higher mean gain would be required to account for the resolution of the channel. Because such high gains per stage do not occur in channels it is evident that the narrow distribution cannot be accounted for statistically, but results from the progressive constraint which space charge imposes on the gain process when the gain exceeds 10^6.

The resolution worsens at gains above 10^8, and as Figure 16 shows, the distribution has a double peak at 5 KV. This effect is due to secondary pulses which appear at the tail of the normal curved channel output pulse. The probability of occurrence of secondary pulses increases with gain and with the length of the straight part of the channel.

The cause of secondary pulsing therefore seems to be ionic feedback. It is not clear why the number of secondary pulses is limited. If the probability of feedback is as high as the pulse height distribution indicates then there ought to be some multiple pulses. All types of curved multipliers produce secondary pulses to some extent, and tertiary and quaternary pulses are also seen, but they are usually very small, so that they do not make appreciable contributions to the pulse height distribution.

The distribution broadens slightly as the repetition frequency increases and the relationship between pulse height and time between pulses becomes more systematic. This is similar to the saturation effect seen in straight channels.

OPERATIONAL CONSIDERATIONS

It is now possible to indicate some operational characteristics of channel multipliers. A channel multiplier will detect any radiation which will excite electrons from the channel wall. Of course the channel entrance may be suitably treated to increase its detection efficiency for certain types of radiation. It is not an energy sensitive device in the sense that a sodium iodide crystal is for example. The pulses from the multiplier signify only that an input event has been detected.

It is possible to make straight multipliers having a sharp pulse height distribution up to tens of kilocycles/sec by using resistive layers of about 10^9 Ω. Straight multipliers have the disadvantage that the gain depends upon gas pressure.

Curved multipliers have a broader pulse height distribution in general, although it is narrower than that obtained from a conventional photomultiplier. The highest detectable count rate is determined by the resistance of the channel and by the noise level in the multiplier-pulse counter system. The choice of resistance is determined by the available power and the need for the channel material to satisfy certain requirements. It must be stable, for example, and it should have a reasonably high secondary emission coefficient. It is necessary to fabricate channels in a variety of shapes and sizes, therefore the channel material must be easily worked. Count rates of the order of 10^5 pulses/sec above a 5 mV threshold are currently available.

Gain maintenance is an important characteristic of channel multipliers but by the nature of the problem, experimental evidence accumulates rather slowly. Deterioration of gain does take place and seems to be due to a combination of material changes and surface contamination.

Because of changes in electron trajectories in channel multipliers it is not possible to operate them in magnetic fields of more than a few hundred gauss.

CONCLUSION

The channel multiplier is a useful research tool for detecting fluxes of energetic particles and radiation up to 10^5 output pulses/sec. At lower count rates it is possible to obtain gains of over 10^9 in straight channels and 10^8 in curved ones. The limitation on gain is probably space charge in curved channels and field distortion effects in straight ones.

ACKNOWLEDGMENTS

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APPENDIX I

Electron Trajectories in Straight Channels

The distance S travelled down the tube between collisions is given by

$$S = \frac{1}{2} E \frac{e}{m} t^2 \qquad \ldots \ldots \ldots (1)$$

E is the axial electric field.
t is the time between collisions.

$$t = d \sqrt{\frac{m}{2eV}} \qquad \ldots \ldots \ldots (2)$$

V is the initial energy in volts, of an electron emitted normally from the channel wall.
d is the channel diameter.

The collision energy, V_c, is given by

$$V_c = E S = \frac{V_o^2}{4 V \alpha^2}$$

where V_o is the total voltage applied to the channel,
and α is the length to diameter ratio.

Let the secondary emission coefficient δ obey the relation

$$\delta = KV_c \qquad K \text{ is a constant}$$

The gain G is given by

$$G = \delta^n$$

where

$$n = \frac{\alpha d}{S}$$

Hence $$G = \left(\frac{KV_o^2}{4V\alpha^2}\right)^{\frac{4V\alpha^2}{V_o}}$$

APPENDIX II

Electron Trajectories in Curved Channels

Consider an electron emitted normally from the outer wall of the channel towards the centre of curvature of the channel, with an initial velocity a. We want to find h, the height of the electron trajectory and the value of θ at which it strikes either wall.

The equations of motion are

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = E\frac{e}{m} \qquad \ldots\ldots (1)$$

$$\ddot{r} = r\dot{\theta}^2 \qquad \ldots\ldots (2)$$

where E, the electric field is parallel to the axis of the channel. In equation (1) $\dot{r}\dot{\theta}$ is always small compared with $\ddot{\theta}$ and can be neglected. The tube diameter d is small compared with R_o and we may assume r is constant and equal to R_o.

Equations (1) and (2) can then be integrated with the initial conditions

$$\dot{r} = -a \qquad r = R_o + d$$

$$\dot{\theta} = \theta = 0$$

the resulting expressions are

$$\dot{r} = \frac{At^3}{3R_o} - a \qquad \ldots\ldots (3)$$

where $A = E\frac{e}{m}$

and $$r = R_o + d + \frac{A^2t^4}{12R_o} - at \qquad \ldots\ldots (4)$$

From (3) and (4) $r_{min} = R_o + d - \frac{3}{4}at$

hence, h the max. height of the electron trajectory is given by

$$h = R_o\left(\frac{81}{16}\beta^2\frac{V^2}{V_o^2}\right)^{1/3} \qquad \ldots\ldots (5)$$

where V is the initial energy of the electron in volts.
V_o is the applied voltage.
β is the ratio of channel length to radius of curvature.
(for a full circle $\beta = 2\pi$)

If we write

$$\frac{\beta R_o}{d} = \alpha,$$

the length to diameter ratio, then from (5) we can show that $h \leqslant d$ if

$$\frac{V_o^2}{V^2} \geqslant \frac{81}{16}\frac{\alpha^3}{\beta} \qquad \ldots\ldots (6)$$

This requirement is fulfilled under most operational conditions.

Since $h < d$, the next collision will be with the outside wall,

and integrating (1) we find

$$R_o\theta_c = \frac{At^2}{2} \qquad \ldots\ldots (7)$$

where θ_c is the angle between collisions.

From (4) we have

$$t^2 = \left(\frac{12a\,R_o}{A^2}\right)^{2/3}$$

hence $$R_o\theta_c = R_o\left(36\frac{V\beta}{V_o}\right)^{1/3} \qquad \ldots\ldots (8)$$

and the energy of collision V_c is given by

$$V_c = \left(36\frac{VV_o^2}{\beta^2}\right)^{1/3} \qquad \ldots\ldots (9)$$

Let us assume that the secondary emission coefficient δ obeys the relation

$$\delta = K\,V_c$$

The gain G is then given by

$$G = \delta^n$$

where n is the number of collisions and

$$n = \frac{\beta}{\theta_c}$$

hence

$$G = \left\{K\left(36\frac{VV_o^2}{\beta}\right)^{1/3}\right\}^{\left(\frac{V_o\beta^2}{36V}\right)^{1/3}} \qquad \ldots (10)$$

APPENDIX III

Probability of Ionisation of Residual Gas

Consider two planes a distance dL apart in the gap of total length L. Electrons are accelerated and may ionise the residual gas. The number of ions found in distance dL is given by

$$dN = np\, f(v)\, dL$$

where dN is the number of ions
n is the number of electrons
p is the pressure in torr
f(v) is the efficiency of ionisation which is a function of V the electron energy.

If the field is uniform

$$\frac{dL}{dv} = \frac{L}{V}$$

$$\therefore \quad dN = np \frac{L}{V} f(v)\, dV$$

The total number of ions formed as an electron accelerates from 0 to V volts in a distance L is then given by

$$N = \frac{npL}{V} \int_0^V f(v)\, dV$$

The function f(v) is given for a number of gases by Meek and Craggs[6] and the corresponding function

$$\frac{1}{V} \int_0^V f(v)\, dV$$

can be obtained numerically. Since its value does not vary widely among a number of the more common gases, one can obtain an estimate of f(v) by assuming the residual gas is oxygen and that the effective ionisation takes place in the last stage of multiplication. Since the energy of the emerging electrons has a peak of about 100 eV, we take V = 100 volts in f(v), and we find its value is about 6. Hence

$$N = 6\, n\, p\, L$$

As an example, if the channel is 10 cm long and the applied voltage is 4000 V then L, the final stage is 0.25 cm. If n, the gain in the starter pulse is 10^6, and the pressure is 10^{-5} torr, then N = 15.

This means that each starter electron pulse produces 15 positive ions. If the gain is higher then of course more positive ions are produced.

Figure 15 shows that one third of all starter pulses did not produce ionic feedback trains under the particular experimental conditions.

The most favourable conditions for feedback are high gain, high pressure and long channels.

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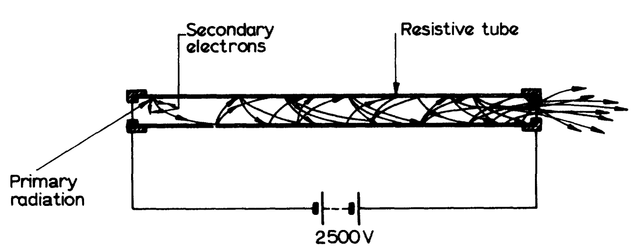

Fig. 1—Channel Electron Multiplier

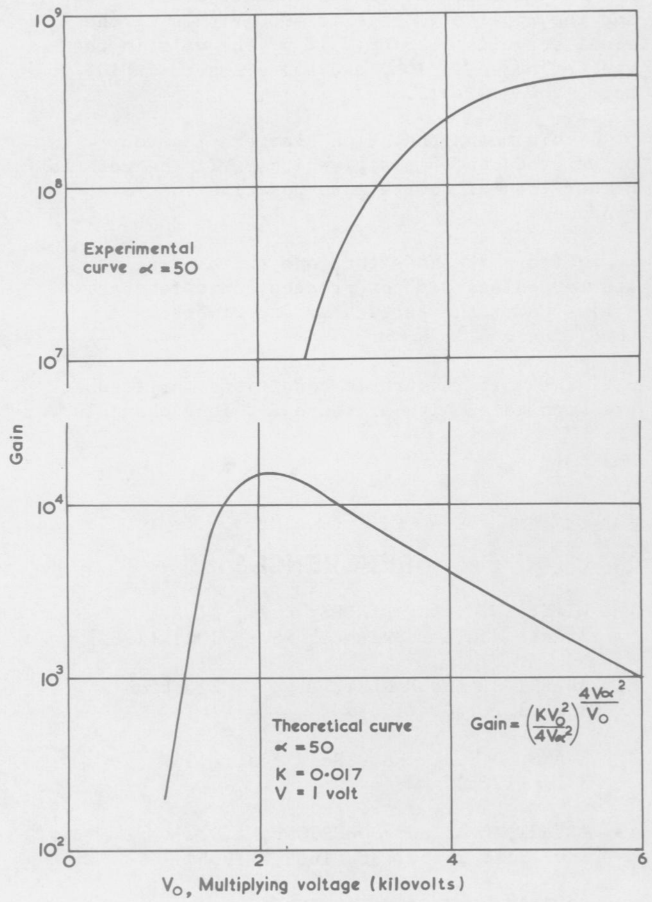

Fig. 2—Comparison of Theoretical and Experimental Gain versus voltage characteristics for a channel multiplier

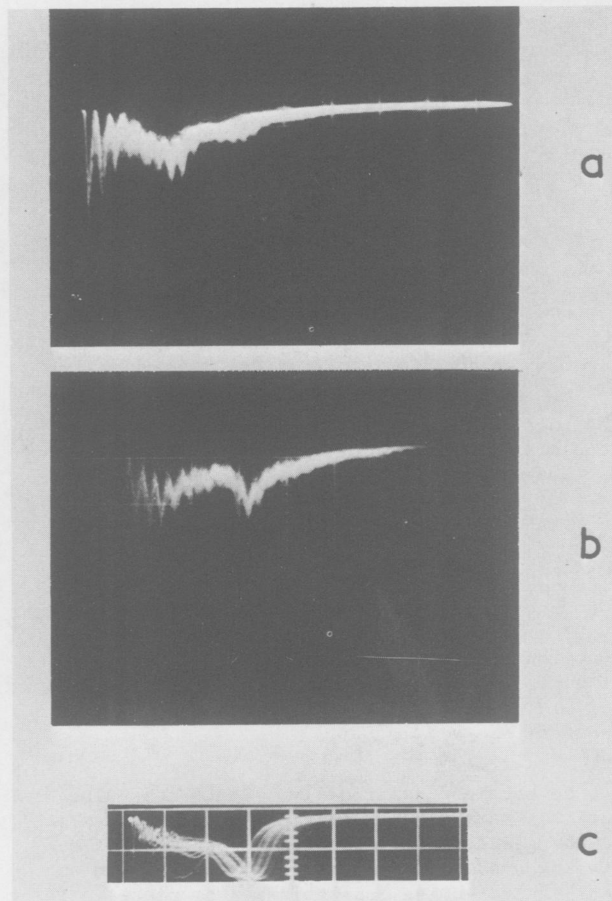

Fig. 4—Ionic feedback pulses produced by (a) argon, (b) xenon, and (c) mercury. Pressure of argon and xenon 4.5 x 10^{-5} torr. Pressure of mercury 8 x 10^{-4} torr. Scale 0.5 μS/large division. The channel used for (a) and (b) was 10 cm long, that for (c) 5 cm.

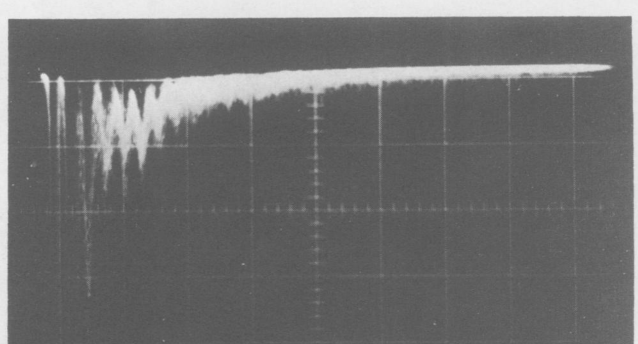

Fig. 3—Ionic feedback pulses from a channel multiplier. Pressure 4 x 10^{-6} torr. Scale 0.5 μsec./large division, 0.01 volts/large division

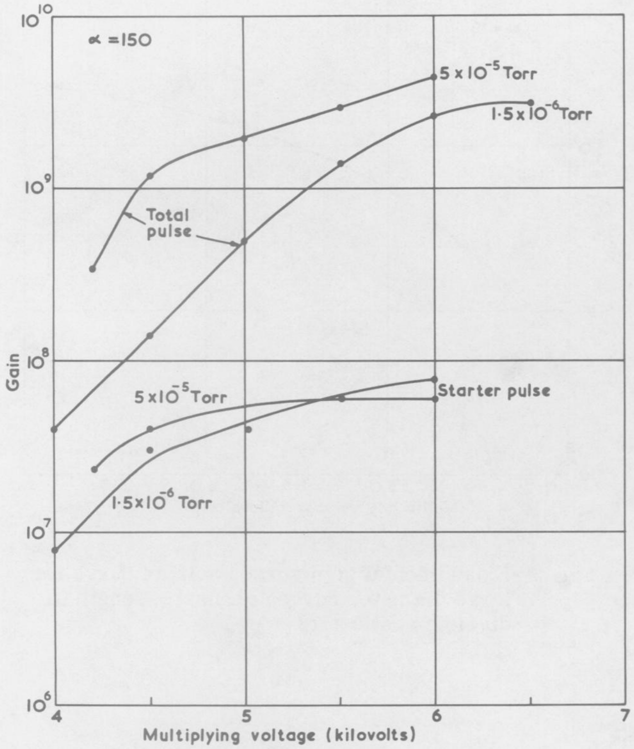

Fig. 5—Gain as a function of voltage in starter pulse and total pulse

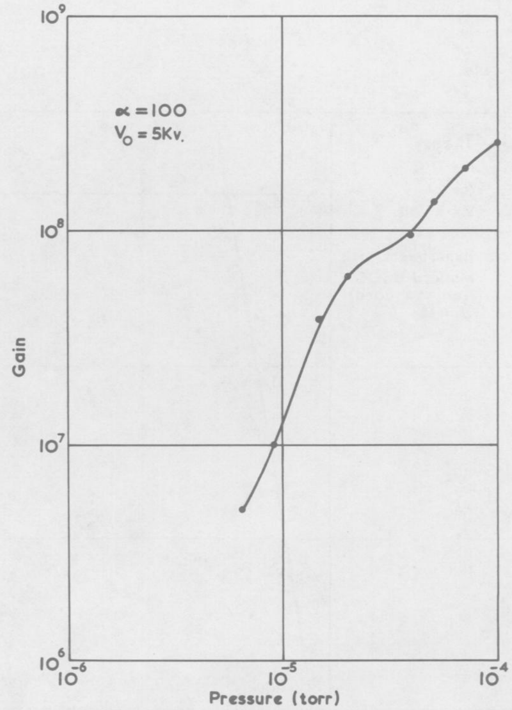

Fig. 6—Gain as a function of pressure at a given voltage

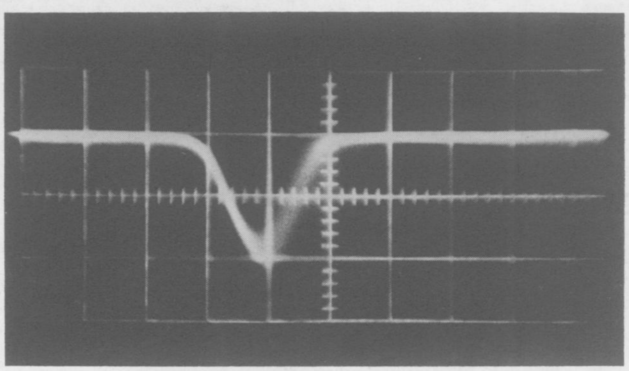

Fig. 7—Output pulse from a curved channel multiplier
Scale 0.02 μS/large division

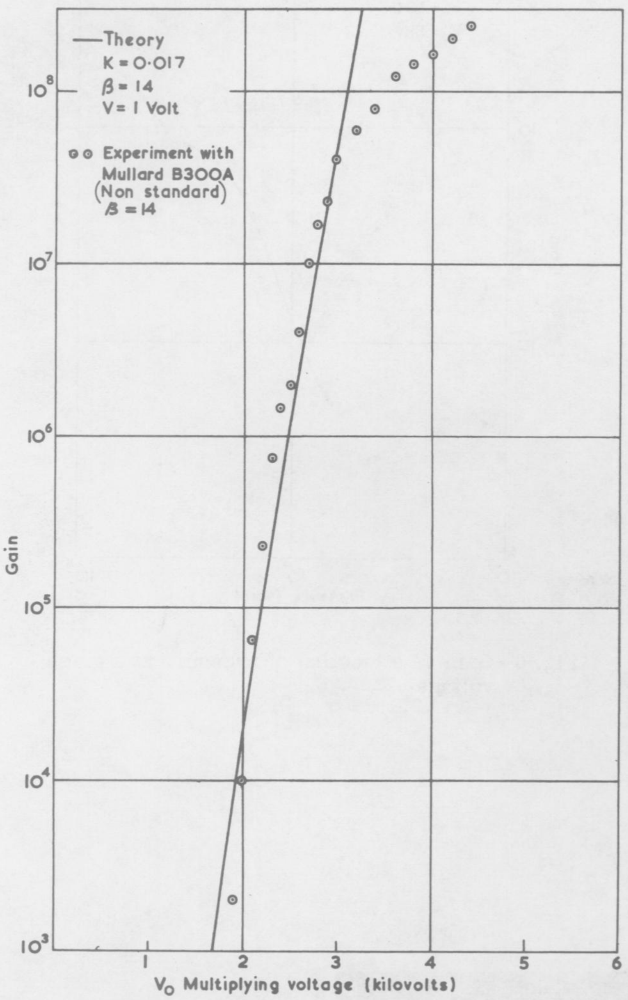

Fig. 8—Theoretical gain versus voltage characteristic for Mullard B300A channel multiplier. Experimental points are also shown

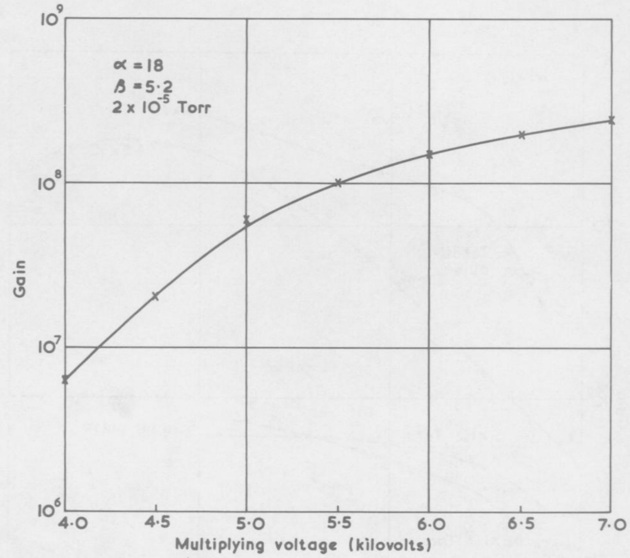

Fig. 9—Experimental gain versus voltage curve for large diameter curved channel. Length to diameter ratio = 18

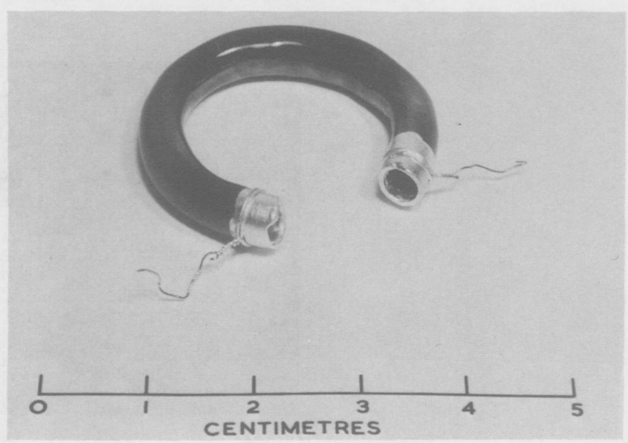

Fig. 10—Large diameter curved channel multiplier. Length to diameter ratio = 18. Angle subtended at centre of curvature = 5.2 radians

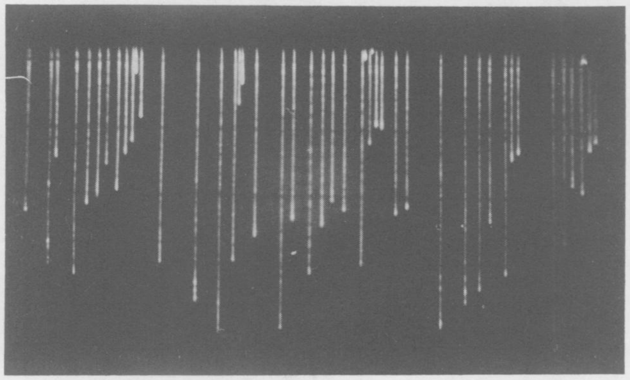

Fig. 11—Output pulses from straight channel illustrating time dependence of pulse height. Applied voltage 7 kV. Pressure 2 x 10^{-5} torr. Gain: about 10^9. Scale 2 msecs./large division. 0.1 volts/large division

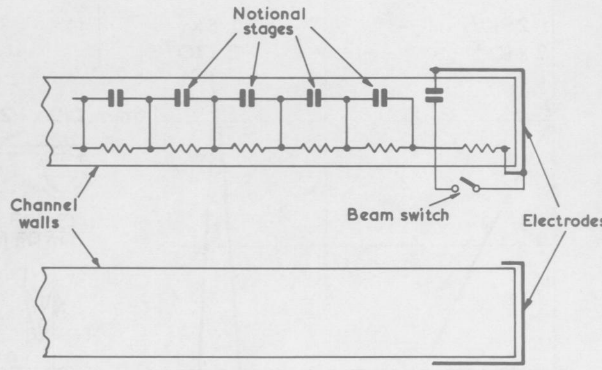

Fig. 13—Equivalent circuit of channel multiplier. The output pulse is obtained when the final capacity is discharged by the "beam switch"

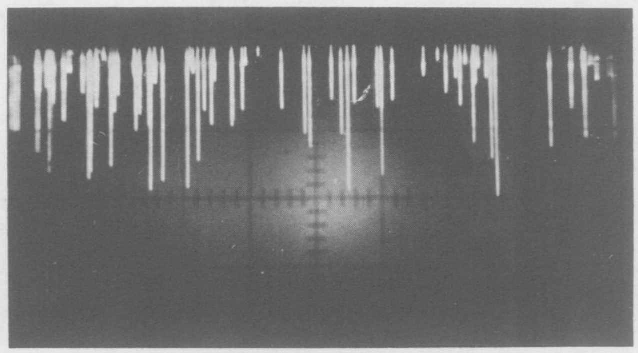

Fig. 14—Output pulses from curved channel showing lack of dependence of height on time. Applied voltage 7 kV, gain about 10^8. Scale 100 msecs./large division, 0.2 volts/large division

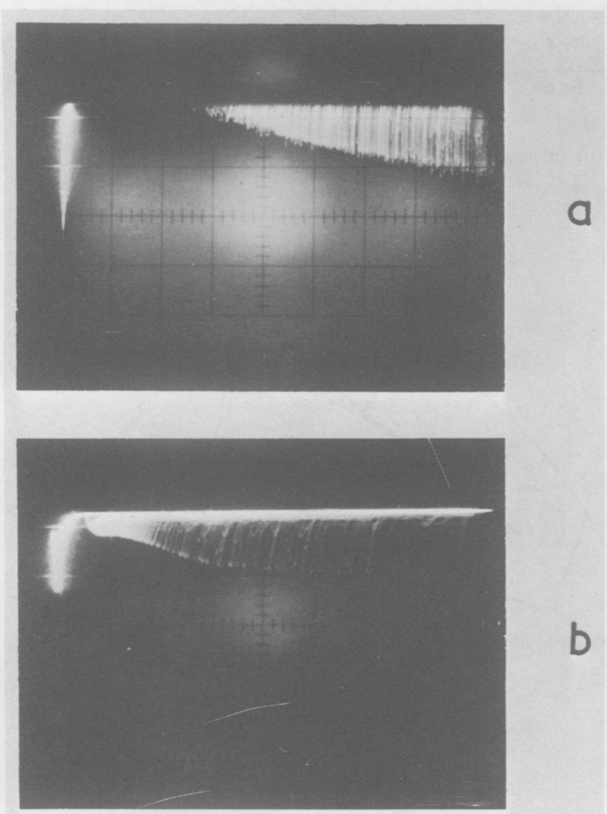

Fig. 12—Effect of channel resistance on recovery time of straight channel multiplier. (a) Resistance 7 x 10^{10} Ω. Time scale 50 μsec./large division. (b) Resistance 10^9 Ω. Time scale 10 μsec./large division

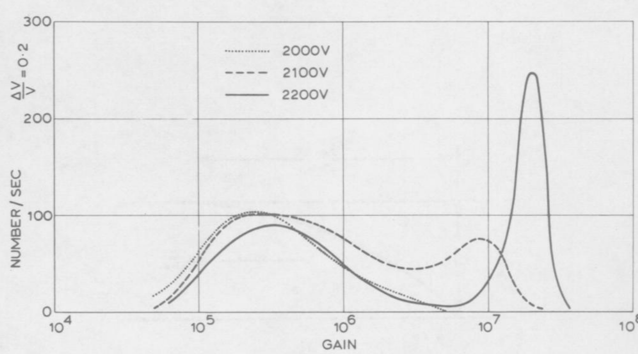

Fig. 15—Pulse height distribution from a straight channel showing distinction between the starter pulse and the total pulse.

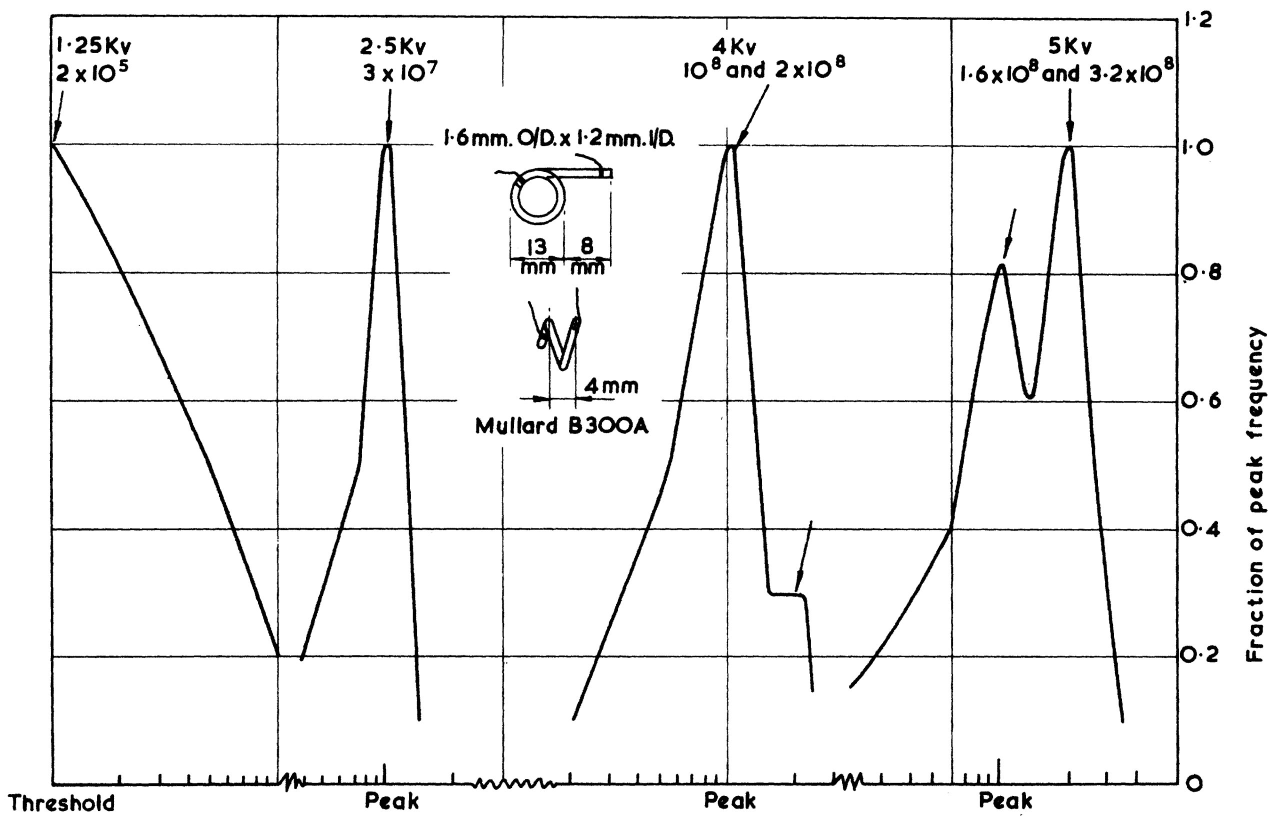

Fig. 16—Pulse height distributions from a Mullard B300A curved channel at various applied voltages. Abscissae logarithmic but not continuous.

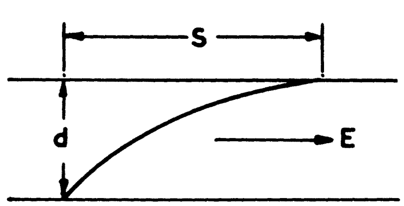

Fig. 17—Electron trajectory in a straight channel

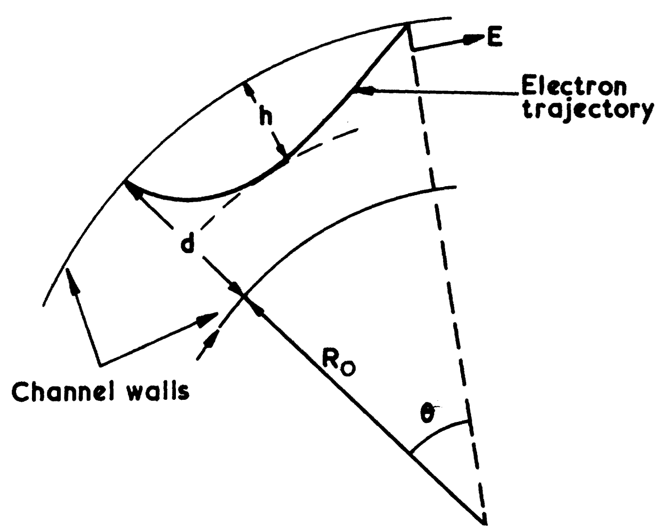

Fig. 18—Electron trajectory in a curved channel

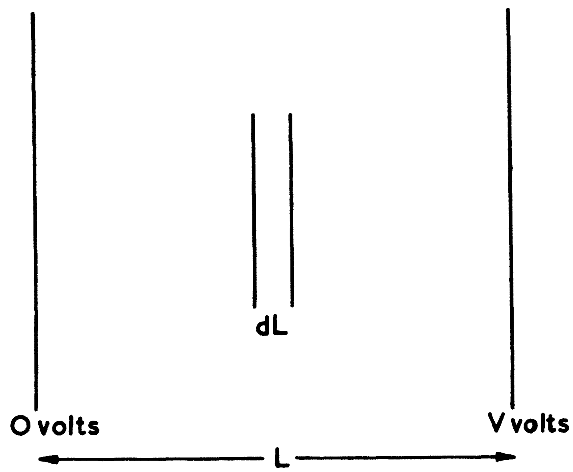

Fig. 19—Gas ionisation by accelerated electrons